Achieving multiple goals via voluntary efforts and motivation asymmetry

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The achievement of common goals through voluntary efforts of members of a group can be challenged by the high temptation of individual defection. Here, two-person one-goal assurance games are generalized to N-person, M-goal achievement games in which group members can have different motivations with respect to the achievement of the different goals. The theoretical performance of groups faced with the challenge of multiple simultaneous goals is analyzed mathematically and computationally. For two-goal scenarios one finds that “polarized” as well as “biased” groups perform well in the presence of defectors. A special case, called individual purpose games (where there is a one-to-one mapping between agents and goals for which they have a high achievement motivation) is analyzed in more detail in form of the “importance of being different theorem”. It is shown that in some individual purpose games, groups can successfully accomplish several goals simultaneously, such that each group member is highly motivated toward the achievement of one unique goal. The game-theoretic results suggest that multiple goals as well as differences in motivations can, in some cases, correspond to highly effective groups. Applying this approach to the case of winemakers making disease control decisions in their respective vineyards shows that game outcomes need not depend on the heterogeneity in the resource value, as previously thought, but they could be more generally driven by motivation asymmetry.

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1. Introduction

Reaching goals represents a key ability of intelligent agents. Reaching a goal in a way that requires the contribution of several agents can be modeled as a game: An assurance game is a game-theoretic model, in which members of a group can choose to spend individual efforts or resources for the achievement of a common goal (Sen and Majumdar, 1969). The choice of exerting an effort toward a goal has a cost (a negative utility), the achievement of the goal has a benefit (positive utility) for each group member. The original formulation of the assurance game corresponds to two agents, one goal and two choices per member of contributing a high effort or a low effort toward that goal. Other names for this class of games are coordination game, trust dilemma or stag hunt (based on a hypothetical scenario proposed by the philosopher Jean-Jacques Rousseau in which two hunters can choose to hunt a stag corresponding to a large payoff or a hare corresponding to a small payoff; the catch is that successfully hunting the stag needs both hunters to choose that option) (McAdams, 2008; Skyrms, 2004).

Such situations can be analyzed within the framework of non-cooperative game theory, in which participants (interchangeably referred to as players, actors, agents or persons) can choose among different actions and strategies in order to maximize their expected outcome. The key concept is that of a (Nash) equilibrium, in which no player can unilaterally improve the outcome by changing the strategy (Nash, 1951).

The classic assurance game comprises three Nash equilibrium points: two pure-strategy equilibria corresponding to mutual cooperation and to mutual defection in addition to one mixed-strategy equilibrium point in which both agents choose between cooperation and defection with probability 1/2.

Milinski et al. studied experimentally iterated assurance games performed by groups with six members, each of whom possesses fine-grained donation options in order to potentially obtain a common-pool reward provided the combined donations of the group are at least as high as a certain threshold (Milinski et al., 2008). Also, it has been noted that the outcome of collective action challenges may depend not only on the rewards but also on the structure of communication networks (Chwe, 2000). In an
inter-species comparison, it has been found that game theoretic considerations influence the decision making in assurance games not only in humans but also in capuchin monkeys (Brosnan et al., 2011). In the case of wild capuchin monkeys, the behavior as well as the payoff structure of achieving the common goal of obtaining prey via hunting in a group versus alone has been shown to depend on the local circumstances such as hunting success rates and access to meat by non-hunters (Boesch, 1994).

The games typically analyzed by game-theoretic analysis deal with scenarios to reach one particular goal (McCain, 2010). Groups are, however, frequently faced with multiple simultaneous challenges: families are challenged with raising children and earning money; societies are challenged with helping those in need, while simultaneously protecting their members from threats. These can be viewed as challenges similar to those found in multi-attribute negotiations, in which participants possess different motivations with respect to different objectives (Lai et al., in press). Multi-attribute game theory has been applied to auctions (Bichler, 2000), border security patrolling (Aguirre et al., 2011) and supply chain network negotiations (Yu et al., 2013).

Because the different choices for the differently motivated participants quickly leads to a “combinatorial explosion” of possibilities, the tractability associated with non-cooperative games can become an issue, thus leading researchers to, for example, “issue-by-issue” analysis approaches of multi-attribute games (Lai et al., in press). An issue-by-issue approach is to tackle one goal at a time. This seems at first sight a plausible strategy, but it eliminates important solutions early on. For example, imagine a situation where there are two different goals $I$ and $II$ and two participants $A$ and $B$ such that solving goal $I$ is important to participant $A$ and achieving goal $II$ is important to participant $B$. Considering both goals with both participants simultaneously may lead to the solution that participant $A$ is tackling goal $I$, while participant $B$ is tackling goal $II$. In contrast, in an issue-by-issue approach, there will be a negotiation for goal $I$ and a separate negotiation for goal $II$. Both negotiations are in danger of breakdown where no goal is achieved, because in each negotiation one participant has low motivation and appears as “defector”.

Here we extend the assumptions underlying assurance games to include those that describe a general achievement game that allows for multiple goals and multiple strategy options per goal for each agent. No requirement is made that the utilities of each agent with respect to each goal are identical (in other words symmetry is not required). Nor is the requirement made that the agents agree on a common strategy (in other words we incorporate a non-cooperative game-theoretic model). This extension to multiple goals is important, because it leads to solutions (i.e. equilibria where goals are achieved) that are not apparent if one considers goals in isolation. The goal of the study is to systematically ascertain how groups consisting of agents with diverse motivations are predicted to perform with respect to multiple simultaneous challenges. This theoretical work may be important for improving the effectiveness of voluntary efforts of groups with respect to achieving non-profit goals. In this study it is explored to what extent multiple simultaneous challenges offer opportunities for goal achievement that are not available in single-goal situations. The need for an improved understanding of such situations is particularly apparent with respect to environmental goals, where a large number of agents with diverse motivations are challenged with prioritizing environmental, economic and other goals.

The importance of being different theorem is presented, that states that a group of $N$ agents faced with achieving $N$ goals, and individual motivations in which each agent is uniquely motivated to spend the effort to solve one particular goal leads to one unique Nash equilibrium point that corresponds to a situation in which all goals are achieved. This theorem reduces – in applicable situations – the need for detailed computer simulations, and facilitates communicability and intuitive understanding of potentially complex game-theoretic situations. These theoretical considerations are augmented by computer results that correspond to group sizes ranging form $2$ to $9$ faced with the challenge of achieving $1$, $2$ or $3$ goals. Both theoretical and computer results indicate that multiple goals and motivation asymmetry can facilitate the achievement of goals without invoking the requirement for iteration or other mechanisms such as reciprocity. Next, an example application is presented in Section 5, where the theory is applied to the case of winemakers and their efforts to invest in disease control measures in their respective vineyards.

The developed approach for achieving goals based on diverse individual motivations can be expected to be rather robust and simple compared to situations where additional mechanisms for achieving compliance are needed. Secondly, the presented solutions are applicable to situations, where only a minority is motivated to achieve certain goals and enforcing mechanisms like coercion are not reliably available because they would need a motivated majority. In other words, the developed theory may have implication for innovative policy-making for cases where traditional approaches have not succeeded. Examples related to viticulture, biodiversity loss and recycling are presented.

2. The multi-goal achievement game

Let there be a scenario in which a group of $N$ agents is faced with the challenge of achieving $M$ different goals. A formal definition of an $N$-agent, $M$-goal achievement game is presented below; note that the function $\Theta : \mathbb{R} \rightarrow \{0, 1\}$ stands for a variant of the Heaviside step function: $\Theta(x) = 1$, if $x \geq 0$ and $\Theta(x) = 0$, if $x < 0$.

Definition 1. Multi-goal achievement game Let there be a set of $N \in \mathbb{N}$ different agents, a set of $M \in \mathbb{N}$ different goals, a number $K \in \mathbb{N}$, $K > 1$ and a cost set $C = \{c_k | k \in \{1, \ldots, K\}\}$, $c_k \in \mathbb{R}$, $c_k \geq 0$, $c_{k_1} < c_{k_2} \iff k_1 < k_2$. Each agent $i \in \{1, \ldots, N\}$ can for each goal $j \in \{1, \ldots, M\}$ choose between spending $d_{ij} \in \mathbb{C}$ currency units toward the achievement of goal $j$. Let $D$ be the $N \times M$ matrix consisting of the elements $d_{ij}$. Let there be an $M$-tuple of positive goal thresholds $T = (g_{1}, g_{2}, \ldots, g_{M}) \in \mathbb{R}^{M}$. We say goal $j$ is achieved, if and only if $\sum_{i=1}^{N} d_{ij} \geq g_j$. Let the utility of agent $i$ be the negative of the sum of payments of agent $i$ plus a sum of rewards obtained via achieved goals, in other words $w_i(D) = -\sum_{j=1}^{M} d_{ij} + \sum_{j=1}^{M} w_i \Theta \left( \sum_{k=1}^{N} d_{kj} - g_j \right)$ with $w_i \in \mathbb{R}$. We call $w_i$ the motivation of agent $i$ with respect to goal $j$. Let $W$ be the $N \times M$ matrix consisting of the elements $w_{ij}$, we call the matrix $W$ the motivation matrix of the game. We call the finite, non-iterative $N$-player game $G(N, M, C, T, W)$ an $M$-goal achievement game or an $N$-player, $M$-goal, $K$-choice achievement game. If $M > 1$, we call the game a multi-goal achievement game, otherwise a single-goal achievement game.

The utility (i.e. total payoff) of an agent is thus the sum of the negative of the chosen payments plus rewards for achieved goals. Note that the reward of an agent $i$ to achieve a particular goal $j$ (represented by motivation matrix elements $w_{ij}$) can consist of a material reward or a subjective motivation or combinations thereof. The motivation matrix elements can nonetheless be measured in currency units (not in terms of financial rewards but in the sense of the willingness to pay for achieving a certain goal).

Motivation is used in this paper as a word for describing the part of the payoffs corresponding to goal achievement not including the chosen payments. The motivation of an agent in connection with one goal can be viewed as the maximum willingness-to-pay of an
agent to reach that goal. Motivation is therefore represented as a measurable “currency” for voluntary efforts.

**Lemma 1.** For an $N$-player, $M$-goal, $K$-choice achievement game, there are $K^{MN}$ different combinations of strategies.

**Proof.** Each player has $K$ choices for each of the $M$ different goals. Each player has thus overall $K^N$ different strategies to choose from. Because there are $N$ different players who can choose their strategy independently, there are $(K^M)^N = K^{MN}$ different combinations of strategies. □

Agents may be motivated to make a payment toward achieving certain goals, but it will not make sense for them to pay more than is needed to achieve the goals that are important to them:

**Lemma 2.** For any agent $i$ and any goal $j$, any strategy $a$ of paying $c_k$ units (with $c_k > g_j$) on goal $j$ is strictly dominated by the strategy $a'$ of paying $c_k$ units on goal $j$ (and unchanged payments toward all other goals $j' \neq j$) if $\exists k' \in \{1, \ldots, k-1\}$ such that $g_k \leq c_k < c_k$.

**Proof.** We will show that for any agent $i$ strategy $a'$ strictly dominates strategy $a$ by showing that the utility $u_i(a')$ of strategy $a'$ is greater than the utility $u_i(a)$ of strategy $a$. First, there is no goal achievement in any goal: $g_j \leq c_k - c_k - g_j - g_j = 0 - \Theta(c_k - g_j) - g_j = 1$. Similarly, one obtains $\Theta(c_k - g_j) - g_j = 1$. In other words, goal $j$ is achieved in either strategy $a$ or $a'$; also the payments toward goals other than goal $j$ are unchanged. It follows that the payoffs are the same for both strategies: $w_{ij} = w_{ij} \cdot \Theta(c_k - g_j) = w_{ij} \cdot \Theta(c_k - g_j) = w_{ij}$. Thirdly, strategy $a$ corresponds to paying $c_k - c_k > 0$ compared to strategy $a'$. The difference in utility to agent $i$ is $u_i(a') - u_i(a) = -c_k + w_{ij} - (c_k - c_k) = -c_k - (c_k - c_k) = 0$. It follows that $u_i(a') > u_i(a)$ in other words for any agent $i$ the utility of strategy $a'$ is greater than the utility of strategy $a$. □

This definition encompasses cases where the difficulty to achieve the different goals varies widely. We define the special case where the goal-achievement thresholds of a multi-goal achievement game are all of the same value as even:

**Definition 2.** Even multi-goal achievement game Let there be a multi-goal achievement game $G(N, M, K, C, T, W)$. If the goal threshold $M$-tuple $T$ is of the form $T = (g, g, \ldots, g) \in \mathbb{R}^M$, we call the game $G$ an even multi-goal achievement game. We call the value of $g$ the universal goal threshold of the game.

We define agents that have an incentive to single-handedly achieve a goal as highly motivated:

**Definition 3.** Let there be a multi-goal achievement game $G(N, M, K, C, T, W)$. We say that an agent $i \in \{1, \ldots, N\}$ has a high motivation with respect to goal $j \in \{1, \ldots, M\}$ if and only if the motivation with respect to goal $j$ is greater than the goal threshold $g_j$ of goal $j$: $w_{ij} > g_j$. We say an agent is highly motivated if there is at least one goal for which the agent has a high motivation.

One focus is on the interesting special case of multi-goal achievement games where the number of agents is equal to the number of goals with the additional provision that there is a one-to-one relationship between the goals and the agents they are highly motivated to achieve. We call such games individual purpose games:

**Definition 4.** Individual purpose game Let there be an achievement game $G(N, M, K, C, T, W)$ where the number of goals $M$ is equal to the number of agents $N$. Let there be a number $L \in \{0, \ldots, N\}$. Let $G$ be as such that the motivation $w_{ij}$ of agent $i$ in $\{1, \ldots, L\}$ for achieving goal $j$ is greater than the goal-achievement cost $g_j$ for $i=j$ and lower compared to $g_j$ for $i \neq j$. For agents $i \in \{L+1, \ldots, N\}$ all motivations are lower than any goal threshold $j \in \{1, \ldots, N\}$: $w_{ij} < \min T$. We call an achievement game with such properties an individual purpose game. We call the case of $L=N$ a complete individual purpose game; the case of $L=0$ is called a defunct individual purpose game, otherwise we call it a partial individual purpose game.

We denote an individual purpose game as $G(N, C, T, W)$, where $C$ is a set of payment options that each agent can pay toward each goal, $T$ is an $N$-tuple of goal-thresholds and $W$ is an $N \times N$ matrix of goal-achievement rewards to an agent’s utility functions.

In some cases, the motivations of the agents toward their non-favorite goals are low; we define such occurrence as extreme individual purpose game:

**Definition 5.** Extreme individual purpose game We say an individual purpose game is extreme if and only if the motivation matrix elements $w_{ij}$ are such that $w_{ij}$ is lower compared to the cost difference between the second-lowest and the lowest-cost action for all agents $i$ and goals $j$ with $i \neq j$, in other words if $w_{ij} < \min \{c_2 - c_1, \ldots, c_j - c_j\}$ for $i \neq j$.

Intuitively, we may suspect that agents in an individual purpose game tend to pay more toward the goals for which they have a high motivation (i.e. $d_i \geq d_i$ for $i \neq j$). Indeed, we are able to show this formally for a special kind of extreme individual purpose game in form of the importance of being different theorem. It turns out that the only equilibrium solution is one where agents pay substantially toward the one goal that is most important to them and nothing toward the goals that are not important to them. While this strategy is highly asymmetric (representing a case of anti-coordination), it has the property that all goals are achieved:

**Theorem 1.** Importance of being different theorem Let there be an extreme even complete individual purpose game $G$ where the lowest payment option $c_1$ to a goal is zero and one of the payment options is equal to the universal goal threshold $g$ of the game. The game $G$ has one and only one Nash equilibrium in which each agent $i$ chooses to pay $g$ units toward goal $i$ and zero units toward all other goals (i.e. $d_i = g$ if $i=j$ and $d_i = 0$ if $i \neq j$). This equilibrium is a pure-strategy equilibrium in which all goals are achieved.

**Proof.** We will show that the described choice of strategies is the only Nash equilibrium of the game by demonstrating that for any agent, this strategy strictly dominates all alternative strategies. We will achieve this by iteratively eliminating dominated strategies. Utilizing Lemma 2, we can rule out strategies where agents pay more than the universal goal threshold $g$ toward any goal, because such strategies are strictly dominated by a strategy where an agent pays exactly $g$ instead of $g' > g$ units toward a goal.

For a strategy $a$ in which agent $i$ pays a cost greater than $c_1$ on a goal $j$, $i \neq j$, that agent can, independently of the choices of the other agents, increase its utility by instead paying $c_1$ units on goal $j$. This is so, because this choice decreases the payment by at least $c_2 - c_1$, it does not change the accomplishments of goal $i$, and it simultaneously decreases the payoff of agent $i$ by a value less than $c_2 - c_1$ due to the potential non-achievement of goal $j$ (since $w_{ij} < c_2 - c_1$ for $i \neq j$). In other words, any strategy other for which $d_i > c_1$ for $i \neq j$ is strictly dominated by strategies for which $d_i = c_1 = 0$ for $i \neq j$ and does not need to be considered further. Furthermore, for a strategy $a$ for which $0 \leq d_i < g$, the utility of agent $i$ is $u_i(a) = -d_i + x_i y_i$, with $x_i = w_i \Theta(\sum_{k=1}^{N} d_i k - g)$ and $y_i = \sum_{j=1}^{M} 1_{j \neq i} w_j \Theta(\sum_{k=1}^{N} d_j k - g)$. Because $d_i = 0$ for $i \neq j$, one obtains $\sum_{k=1}^{N} d_i k = d_i$ and $\sum_{k=1}^{N} d_j k = d_j$. It follows that $x_i = w_i \Theta(d_i - g)$ and $y_i = \sum_{j=1 \neq i}^{M} w_j \Theta(d_j - g)$. Because $d_i \leq g$, it follows that goal $i$ is not achieved (because $\Theta(d_i - g) = 0$) and $x_i = 0$. Thus $u_i(a) = -d_i + y_i$. If the payment $d_i$ paid by agent $i$ on goal $i$ is, on the other hand, equal to $d_i = g$ (strategy $a'$), the utility for agent
i is \( u_i(a') = -d_i + w_i \Theta(d_i - g) + y = -g + w_i + y \). Because \( w_i > g \), it follows that \( u_i(a') > y = u_i(a) \). In other words, the choice of agent \( i \) paying \( g \) units on goal \( i \) strictly dominates the choice of paying less than \( g \) units on goal \( i \). This leaves for each agent \( i \) exactly one strictly dominating strategy of paying \( g \) units on goal \( i \) and zero units on all goals other than goal \( i \). From this it follows that the strategy profile of each agent \( i \) paying \( g \) units on goal \( i \) and zero units on all goals other than goal \( i \) corresponds to the one and only one Nash equilibrium of the game. The identified Nash equilibrium represents a pure-strategy equilibrium. Because \( g \) units are paid toward each goal \( i \), all goals are achieved. \( \square \)

A scenario that illustrates this situation is one in which a group of \( N \) children is asked to feed and groom \( N \) pet animals. If one child chooses to not put in the required effort, the other children might be tempted to also stop working toward the upkeep of the group of pet animals. If, on the other hand, a child is “in love” with a pet animal, it will keep up the maintenance of that animal no matter how the other children are acting. If each child is “in love” with one unique animal, all \( N \) animals are being cared for such that each child takes care of the pet animal it is “in love” with, while not contributing to the maintenance of the other pet animals. An analogous approach pertaining to ecology can be obtained from environmental non-profit organizations that ask for donations toward individualized environmental objectives such as “adopting” an acre of rainforest as opposed to donating toward an unspecific pool of resources that is used for many objectives (Igoe, 2010).

We can generalize this theorem for the case of partial or defunct individual purpose games where there are some agents that are not motivated to individually achieve any goals. One finds that such unmotivated agents choose to not make payments toward any goals. Interestingly, the goals for which there are agents that have a high incentive to achieve them keep being achieved even if there are agents that choose to not make any payments. Note that in this paper we use the convention that a set described in roster notation corresponds to the empty set if the stated lower bound is higher than the upper bound: \( \{A, \ldots, B\} = \emptyset \) if and only if \( A > B \):

**Theorem 2.** Generalized importance of being different theorem Let there be an \( N \) – goal, \( N \) – agent extreme even individual purpose game \( G \) where the lowest payment option \( c_i \) to a goal is zero and one of the payment options is equal to the universal goal threshold \( g \) of the game. Let \( L \in \{0, \ldots, N\} \) be the number of agents that are highly motivated, and \( w_i > g \), for \( i \in L \) and \( w_i < g \), for \( i \geq L \). The game \( G \) has one and only one Nash equilibrium in which each agent \( i \in \{1, \ldots, L\} \) chooses to pay \( g \) effort units toward goal \( i \) and zero effort units toward all other goals \( \{d_j = g \text{ if } j = i \text{ and } d_j = 0 \text{ if } j \neq i\} \); agents \( i \in \{L + 1, \ldots, N\} \) choose to not make any payments. This equilibrium is a pure-strategy equilibrium in which \( L \) goals are achieved.

In other words, this generalized scenario of only a subset of agents being highly motivated is resilient with respect to the presence of defectors that have zero motivation with respect to any goal: the \( L \) goals for which the highly motivated agents have an incentive to achieve keep being attained. Importantly, Theorem 2 pertains to \( L \leq N \) highly motivated agents is a generalization of Theorem 1, because Theorem 1 is identical to Theorem 2 for the special case of \( L = N \).

**Proof.** We will show that the described choice of strategies is the only Nash equilibrium of the game by demonstrating that for each agent, this strategy strictly dominates all alternative strategies. We will achieve this by iteratively eliminating dominated strategies.

Utilizing Lemma 2, we can rule out strategies where agents pay more than the universal goal threshold \( g \) toward any goal, because such strategies are strictly dominated by a strategy where an agent pays exactly \( g \) units toward a goal.

Strategies for which not highly motivated agents \( i \in \{L + 1, \ldots, N\} \) make non-minimal payments are strictly dominated by strategies corresponding to minimal (if possible zero) payments. This is so, because for such agents \( i, w_i = 0 \) and thus their utility as a function of their payment choices \( D = \sum_{j=1}^{M} d_j. \) This utility is maximized if the chosen payments \( d_j \) are minimal. Because the lowest available payment option is zero, agents \( i \in \{L + 1, \ldots, N\} \) will choose to not make any payments toward any goals.

For a strategy in which agent \( i \in \{1, \ldots, L\} \) pays a cost greater than \( c_1 \) on a goal \( j, i \neq j \), that agent can, independently of the choices of the other agents, increase its utility by instead paying \( c_1 \) units on goal \( j \). This is so, because this choice decreases the payment by at least \( c_2 - c_1 \), it does not change the accomplishments of goal \( i \), and it simultaneously decreases the payoff of agent \( i \) by a value less than \( c_1 \) due to the potential non-achievement of goal \( j \) (since \( w_i < c_2 - c_1 \) for \( i \neq j \)). In other words, any strategy for which \( d_j > c_1 \) for \( i \neq j \) is strictly dominated by strategies for which \( d_j = c_1 \). If \( i \neq j \) and does not need to be considered further.

For a strategy \( a \) for which \( 0 < d_i < g \), the utility of agent \( i \in \{1, \ldots, L\} \) is \( u_i(a) = -d_i + x + y \), with \( x = w_i \Theta \left( \sum_{k=1}^{N} d_k - g \right) \) and \( y = \sum_{j=1}^{N} d_j - g \). Because \( d_j = 0 \) for \( i \neq j \), one obtains \( x = w_i \Theta \left( \sum_{k=1}^{N} d_k - g \right) \) and \( y = \sum_{j=1}^{N} d_j - g \). It follows that \( x = w_i \Theta \left( \sum_{k=1}^{N} d_k - g \right) \) and \( y = \sum_{j=1}^{N} d_j - g \). Because \( d_i < g \), it follows that goal \( i \in \{1, \ldots, L\} \) is not achieved (because \( \Theta(d_i - g) = 0 \)) and \( x = 0 \), thus \( u_i(a) = -d_i + y \leq x \). If the payment \( d_i \) paid by agent \( i \in \{1, \ldots, L\} \) on goal \( i \) is, on the other hand, equal, to \( d_i = g \) (strategy \( a' \)), the utility for this agent \( i \) is \( u_i(a') = -d_i + w_i \Theta(d_i - g) + y = -g + w_i + y \). Because \( w_i > g \), it follows that \( u_i(a') > y = u_i(a) \). In other words, the choice of agent \( i \) paying \( g \) units on goal \( i \) strictly dominates the choice of paying less than \( g \) units on goal \( i \). This leaves for each agent \( i \in \{1, \ldots, L\} \) exactly one strictly dominating strategy of paying \( g \) effort units on goal \( i \) and zero effort units on all goals other than goal \( i \). From this it follows that the strategy profile of each agent \( i \in \{1, \ldots, L\} \) paying \( g \) units on goal \( i \) and zero units on all goals other than goal \( i \) and agents \( i \in \{L + 1, \ldots, N\} \) making any payments corresponds to the one and only one Nash equilibrium of the game. The identified Nash equilibrium represents a pure-strategy equilibrium. Because \( g \) units are paid toward each goal \( i \in \{1, \ldots, L\} \), \( L \) goals are achieved. \( \square \)

**3. The computational approach**

The developed theorems are applicable to special situations such cases where there is a one-to-one mapping between goals and agents that are highly motivated to achieve them. Real-world scenarios are oftentimes more complex. To better understand the range of possible scenarios and their predicted outcomes, a computational model was developed. This model contains additional interesting features (explained in more detail below) such as the possibility that the difficulty of achieving goals may be subject to variability or groups may contain “black sheep” of members that are not motivated to contribute toward any goals.

Variants of goal achievement games are used for the computational results, in which each agent has for each goal the choice of paying \( 0, \frac{1}{2} \) or 1 units (payments). These three choices are called defection, cooperation and heroic effort respectively. The threshold for goal achievement is for each goal set to \( N/M \); in other words, a specific goal is achieved, if the total payments toward that goal is at least \( N/M \). To achieve all goals, the sum of all payments of the agents has thus to be at least \( N \) units. The goal-specific utility of agent \( i \) with respect to goal \( j \) is defined as described in Definition 1. The matrix elements are for our numerical analysis set equal to one of the costs of the three possible choices plus a small excess motivation term
δ (set to 0.25 utility units). The rationale for the excess motivation term δ is twofold: first, it prevents misassignments of equilibria due to rounding errors. Secondly, the term prevents ambiguities of identical total payoffs for two strategies for cases where the cost to reach a goal is exactly equal to the payoff of reaching this goal. These scenarios correspond to generalized achievement games and are analyzed using a game-theoretic approach.

Four different performance scores are defined that measure how well a group is able to achieve goals. These four scores are:

- The mean–goal-achievement score (MGA) is the mean of achieved goals, averaged over the different Nash equilibria of the achievement game.
- The all-goal-achievement score (ALL) score for one Nash equilibrium of an achievement game is equal to 1 if all goals are achieved and zero otherwise. The ALL score of a game is the average over the ALL scores of all equilibrium points.
- The deflection-robustness score (DD) is the average of the MGA scores over all possible scenarios, in which exactly one agent is replaced by an agent with low motivation (set to δ) toward all goals. This score measures the robustness of goal achievement under the challenge of additional defectors.
- Variable-load score (VL) is the MGA score of an achievement game average over all scenarios in which the threshold to achieve one goal is instead of n set to either n + 1 or n − 1 (with n being equal to the number of agents). This score measures the robustness in goal achievement with respect to the challenge of a variable difficulty in achieving each goal.

We define a measure of group “polarization”: The divergence of two agents is the squared Euclidean norm of the difference of their motivation M-tuples divided by the number of goals. The divergence V of a group is defined as the maximum divergence of any pair of its agents:

\[ V = \frac{1}{M} \max_{i,j \in \{1, \ldots, N \}} \sum_{k=1}^{M} (w_{ik} - w_{jk})^2 \]

Computational results were generated examining numerically identified pure-strategy Nash-equilibrium points of achievement games and iterating over group sizes ranging from 2 to 5 and over the number of goals ranging from 1 to 3. The analysis of an N-player M-goal achievement game has been implemented with the use of the Java programming language.

The algorithm works as follows. For a given group size and number of goals, the program iterates over all possible action choices of each agent. For any given combination of choices of the agents, the program determines the total payments for each goal and which goals are achieved. Once it is determined which goals are achieved, the payoffs for each agent are computed. Next, it is determined whether the agents’ choices correspond to a Nash equilibrium (i.e. if no agent can improve his/her payoff by unilaterally changing the strategy). The program performs a complete enumeration of all combination of choices of all agents with respect to all goals, and determines which of the choice combinations correspond to Nash equilibria. Once all Nash equilibria are determined, the different group performance scores are computed. The computer program performs the creation of different group scenarios (by iterating over different types of agent motivations), the identification of pure-strategy Nash equilibria and the computation of the different scores. Mixed-strategy equilibria are not considered.

Practically, results corresponding to a provided number of agents and number of goals can be obtained by calling a wrapper script within a Unix terminal environment (such as Linux or Apple OS X) as follows:

```
./coordinate.sh NUMBERGOALS NUMBERAGENTS
```

The Java sources of the program are available at https://bitbucket.org/solace/jenvirocon.

Group sizes of 2–5 members faced with the challenge of achieving one, two or three goals have been analyzed. The case of a group with five members faced with the achievement of three goals has not been analyzed numerically, because of its large computational cost of iterating over all possible group member motivations and payment choices.

4. Results

One can define for each agent the sum of all motivations toward all goals. The sum of the total motivations leads, in turn, to the total motivation of a group, as a measure of it’s capability to achieve goals. Dividing that number by the number of group members leads to a quantity that makes groups of different sizes comparable (called here “mean motivation”). The mean motivation is defined as the payoff for achieving goals (not including the payments for achieving those goals), averaged over all goals and all group members. Fig. 1 depicts the mean-goal-achievement score as a function of a group’s mean motivation for different numbers of agents and numbers of goals. As expected, the mean-goal-achievement score tends to be higher for groups with higher mean motivation. The groups are considered “divergent” or “non-divergent”, depending on whether or not the magnitude of their maximum difference in goal priorities (see definition of divergence V) is greater than a threshold value. This threshold was chosen to be 0.5 because it is the midpoint of the range [0, 1] of possible divergence scores V that groups can attain. One can see that for cases in which the number of agents is equal to the number of goals (2 and 3), the divergent groups tend to outperform the non-divergent groups.

Differences in goal achievement between divergent and non-divergent groups are depicted in Fig. 2. Figs. 1 and 2 demonstrate that for the case of 2 goals and more than 2 agents one can identify regions, where divergent groups tend to do better, worse, or similar to non-divergent groups.

Scatter plots for the MGA-score and for the three alternative performance scores (termed ALL, DD and VL) are shown in Fig. 3. One can see that overall tendencies for the alternative scores are not dramatically different compared to the MGA scores depicted in Fig. 1: in the shown plots, one can observe an increase in a roughly sigmoidal fashion of the performance (i.e. goal achievement) of a group as a function of its mean motivation to achieve those goals. The dependency of goal achievement on the precise composition of which group members are motivated to achieve which goals is weak for groups with an overall very high or very low mean motivation and strong otherwise.

In Tables 1–3 the different scores of groups consisting of two, three and four members, respectively, that are faced with two goals are listed. Each agent can be one of three types: A stands for a high motivation with respect to goal 1, and a low motivation with respect to goal 2. B stands for agents who have a high motivation with respect to goal 2 and a low motivation with respect to goal 1. Type O stands for agents, who have a medium motivation (i.e. 0.5) with respect to each of the two goals. Note that the polarization of a group is for Tables 1–3 ascertained via inspection of its member motivations as opposed to a formal score. Shown are the mean-goal-achievement score (MGA), the all-goal-achievement score (ALL), the deflection-robustness score (DD) and the variable-load-score (VL) as well as their corresponding ranks (indicated as
Fig. 1. Shown is the mean-goal-achievement (MGA) score as a function of mean motivation for groups consisting of two to five members faced with the achievement of two or three goals. The groups have been split in “divergent” and “non-divergent” groups, corresponding to whether they have a high or low maximum difference in goal priorities between any two group members. The mean-goal-achievement score is the fraction of goals that are achieved averaged over the pure-strategy Nash equilibria. A corresponding scatter plot that includes the case of one goal is shown in Fig. 3.

Table 1
2-Goal, 2-member achievement game, in which each member can be of type A,B or O, corresponding to motivations toward goals 1 and 2 being \( (1 + \delta, \delta, 1 + \delta), (0.5 + \delta, 0.5 + \delta) \) respectively (with \( \delta = 0.25 \)). MGA: the score assigned to a group is the fraction of achieved goals averaged over the equilibrium points. ALL: number of equilibria in which all goals are achieved divided by the total number of equilibrium points; DD: the MGA score, averaged over groups in which the motivation of one group member is set to \( (\delta, \delta) \). VL: the average of the MGA score for which the threshold to achieve one goal is one unit higher or one unit lower; MGA, ALLR, DDR, VLR: the rank of group among the listed groups with respect to the MGA, ALL, DD or VL score, respectively. Tied ranks are replaced with the minimum tied rank. Wins: number of cases, in which a group has better ranks with respect to more scores compared to another group. Ties: number of cases, in which a group has tied score ranks compared to another group. One can see that group AB is outperforming (or tied with) the other groups with respect to all scores. In other words, groups of two, in which one member is highly motivated with respect to one goal, while the other group member is highly motivated with respect to the other goal are more likely to achieve both goals (ALL score of 1.0) compared to the other listed groups.

Table 2
Performance of groups of 3 members faced with a challenge of achieving two goals and having equal total motivations.

<table>
<thead>
<tr>
<th>Motivations</th>
<th>MGA</th>
<th>ALL</th>
<th>DD</th>
<th>VL</th>
<th>MGA</th>
<th>ALLR</th>
<th>DDR</th>
<th>VLR</th>
<th>Wins</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOB</td>
<td>0.50</td>
<td>0.25</td>
<td>0.17</td>
<td>0.50</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>7.00</td>
<td>2.00</td>
</tr>
<tr>
<td>OOO</td>
<td>0.50</td>
<td>0.25</td>
<td>0.00</td>
<td>0.50</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>3</td>
<td>6.00</td>
<td>2.00</td>
</tr>
<tr>
<td>AAA</td>
<td>0.44</td>
<td>0.00</td>
<td>0.33</td>
<td>0.44</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>6.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>BBB</td>
<td>0.44</td>
<td>0.00</td>
<td>0.33</td>
<td>0.44</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>6.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>OBB</td>
<td>0.42</td>
<td>0.00</td>
<td>0.28</td>
<td>0.52</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>4.00</td>
<td>3.00</td>
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</tr>
<tr>
<td>AAO</td>
<td>0.42</td>
<td>0.00</td>
<td>0.28</td>
<td>0.52</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>4.00</td>
<td>3.00</td>
<td></td>
</tr>
<tr>
<td>AOO</td>
<td>0.38</td>
<td>0.00</td>
<td>0.17</td>
<td>0.44</td>
<td>7</td>
<td>3</td>
<td>5</td>
<td>2.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>OOB</td>
<td>0.38</td>
<td>0.00</td>
<td>0.17</td>
<td>0.44</td>
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<td>3</td>
<td>5</td>
<td>2.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>OBB</td>
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<td>0.00</td>
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<td>0.42</td>
<td>9</td>
<td>3</td>
<td>8</td>
<td>9</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>AAB</td>
<td>0.33</td>
<td>0.00</td>
<td>0.11</td>
<td>0.42</td>
<td>9</td>
<td>3</td>
<td>8</td>
<td>9</td>
<td>0.00</td>
<td>1.00</td>
</tr>
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</table>

MGAR, ALLR, DDR and VLR respectively. The low DD score of 0.0 for the OOOO group (a group consisting of four “centrist” members) that have medium motivation toward achieving each of the two goals) indicates that a ‘centrist’ group is doing relatively well under regular conditions (an MGA-score value of 0.5) but is highly vulnerable to the introduction of an agent with low motivation that leads to a break-down of goal achievement. The semi-polarized (AOOB) and polarized (AABB) groups perform well under unchallenged conditions (MGA score rank 1 tied with the OOOO group) but perform also well under conditions in which the difficulty of reaching goals is fluctuating (the two groups have the highest-ranking variable.
Fig. 2. Top-performing divergent groups frequently outperform top-performing non-divergent groups, especially if the number of goals is equal to the number of group members. Shown is the difference in mean-goal-achievement (MGA) score of top-performing divergent and non-divergent groups as a function of total group motivation. A positive value indicates that top-performing divergent groups score higher compared to top-performing non-divergent groups. For the analysis, intervals with a width of 0.1 with respect to a group’s mean motivation have been chosen. For each interval, the difference between the MGA score of the top-performing divergent group and the top-performing non-divergent group has been computed. The width of the ribbon is computed as the sum of the median absolute deviations (mad) values of the MGA scores for the compared sets of divergent and non-divergent groups.

Table 3
Performance scores of groups with 4 members that are faced with the challenge of achieving two goals (called goal 1 and 2). Each group member can be of type A, B or O corresponding to a high (low), low (high) or medium (medium) motivation with respect to goal 1 (2). The score definitions are described in the caption of Fig. 1. One can see, that the (semi) “polarized” groups AABB and AOOB score well in comparison to the “balanced” group OOOO. Surprisingly well perform groups AAAA and BBBD that essentially achieve one goal reliably while ignoring the other goal.

<table>
<thead>
<tr>
<th>Motivations</th>
<th>MGA</th>
<th>ALL</th>
<th>DD</th>
<th>VL</th>
<th>MGA</th>
<th>ALL</th>
<th>DD</th>
<th>VL</th>
<th>Wins</th>
<th>Ties</th>
</tr>
</thead>
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<td>1</td>
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<tr>
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<td>AAOO</td>
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<td>2.00</td>
<td>1.00</td>
</tr>
<tr>
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<td>1.00</td>
</tr>
</tbody>
</table>

5. Application: plant disease control based on voluntary efforts

We can apply this theoretical approach to a known management problem in environmental economics and ecology of winemakers contributing efforts to control plant diseases in their respective vineyards.
Fig. 3. Shown are different goal achievement scores as a function of mean motivation for groups consisting of two to five members faced with the achievement of one, two or three goals. The color indicates the “divergence”, in other words a measure of how similar the motivations are between group members. Blue indicates homogenous groups (divergence score of zero), red indicates heterogeneous groups (divergence score of 1). Top left: mean-goal-achievement score (the fraction of goals that are achieved averaged over the pure-strategy Nash equilibria). Top right: all-goal score (the all-goal score is the average is the fraction of Nash equilibria in which all goals are achieved). Bottom left: variable-load score (the average in mean goal achievement for all scenarios for which one goal has one unit higher cost of achievement and all scenarios in which one goal has one unit lower cost of achievement). Bottom-right: defection-robustness score (the average in mean goal achievement for all scenarios for which one group member is replaced by an agent who has low motivation to achieve any goals). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Grapevine diseases affect grape harvests all over the world (Fuchs et al., 2009; Golino et al., 2008; Martin et al., 2005; Cabaleiro et al., 2008; Charles et al., 2009; Martelli and Boudon-Padieu, 2006). The grapevine leafroll disease (GLRD) has recently received particular attention because it reduces both grape yield and wine quality. GLRD delays fruit ripening, lowers soluble solids, increases fruit acidity, reduces tannin content, and causes undesirable flavor profiles (Goheen et al., 1959; Martelli and Boudon-Padieu, 2006; Martinson et al., 2008; Naidu et al., 2014).

The economic impact of GLRD was estimated at $25,000–$40,000 per hectare in New York State vineyards (Atallah et al., 2012) and $29,900–$226,400 per hectare in California (Ricketts et al., 2015). GLRD is primarily introduced via vegetative propagation and is transmitted from infective to healthy vines by several species of mealybugs (Hemiptera: Pseudococcidae) and soft-scale insects (Hemiptera: Coccidae) (Martelli and Boudon-Padieu, 2006; Pietersen, 2006; Tsai et al., 2008).

The control of insect-transmitted plant diseases such as GLRD consists of reducing the population of disease vectors (Ricketts et al., 2015) and minimizing secondary sources of infection by roguing (removing) infected plants and replacing them with healthy ones (Atallah et al., 2012; Chan and Jeger, 1994). Atallah et al. (2015) focus on non-pesticidal management strategies and evaluate nonspatial and spatial roguing strategies according to vineyard expected net present values (rewards minus control costs). Nonspatial strategies consist of roguing vines based on their symptomatic infection state and their age. Spatial strategies take advantage of the spatial disease ecology: a manager roges and replaces a symptomatic vine but also performs a virus test on non-symptomatic vines that are located in its neighborhood and then roges them and replaces them if they test positive. They find that spatial strategies targeting immediate neighbors of symptomatic vines dominate nonspatial strategies. Atallah et al. (2017) consider the case of GLRD control in two vineyards producing low and high quality wines that are ecologically-connected (through insect disease vectors) and independently managed. In their model, one vineyard produces high-value wine grapes whereas the other produces low-value wine grapes. They consider the above-stated control strategies in addition to buffer zones strategies that consist of removing (without replanting) vines in the border columns of a vineyard in order to reduce long-distance disease diffusion between vineyards. They find that, if the managers agree to cooperatively control the disease, the Nash bargaining solution consists of both spatially controlling the disease after side-payments. However, in a noncooperative setting, they find a unique Nash equilibrium pair of strategies that consists of no control in either vineyard.

We consider two winemakers 1 and 2 with two respective vineyards. We define two goals G1 and G2 corresponding to the goal of implementing disease control measures in vineyard 1 and 2, respectively. We define g1 as the cost to reach G1 and g2 as the cost to reach G2. We initially assume that the vineyards are of equal size and that that produced wine is of virtually equal quality in each vineyard. This leads to equal costs of control measures in each vineyard: g1 = g2 = g. In other words, this is according to the presented definitions an even game with all goal thresholds being equal. We consider both cases of the two vineyards being ecologically connected (i.e. disease can be transmitted between them) or not connected.

We define a motivation matrix that represents contributions to each agent’s utility not including the disease control cost. We assume, that each winemaker has for each vineyard (even the competitor’s vineyard) the options to implement or to not implement disease control measures. In other words, the lowest cost action is to pay 0 toward G1 and 0 to G2. The other available action is to implement disease control measures and to pay g toward G1 and/or g toward G2.

The motivations (i.e. rewards not including payments) for achieving the goals are assumed to be:

- w11: reward of agent 1 to achieve goal 1: R1 = r1 + s1
- w22: reward of agent 2 to achieve goal 2: R2 = r2 + s2
- w12, w21: reward of agent 1 to achieve goal 2 or vice versa

With r1 and r2 corresponding to the financial reward of implementing disease control for each winemaker and s1 and s2 corresponding to a potential subjective reward. Note that s1 and s2 do not have to be equal: they represent variations in the individual motivations of winemakers 1 and 2. This could be due, for example, to one winemaker being motivated by a profit-maximizing goal (corresponding to s = 0) and another winemaker having a higher environmental motivation of avoiding disease (corresponding to s > 0).

The motivation of agent 1 to achieve goal 2 and agent 2 to achieve goal 1 is x. Let us assume that x is smaller than g: this is realistic because while an adjacent vineyard being disease free is desirable it is not as important as the own vineyard being disease free. A value of x > 0 corresponds to vineyards that are ecologically connected, because it is advantageous if an adjacent vineyard is disease free. The case of two vineyards that are not ecologically connected corresponds to x = 0.

The following cases are particularly interesting:

- Case 1: R1 > g and R2 > g: Because the number of goals is equal to the number of agents, and the contribution to the utility is greater than the goal thresholds for i=1 and x<g for i ≠ j the conditions for an individual purpose game are fulfilled. This is also an extreme individual purpose game because wij (i ≠ j) is x and thus lower than the difference between the lowest cost action (0) and the second lowest cost action (g) (this difference is g − 0 = g > x).

Table 4: Payoff matrix corresponding to 2 winemakers and 2 vineyards. Each winemaker has 4 possible actions of paying toward disease control in (i) no vineyard, (ii) vineyard 1, (iii) vineyard 2 or (iv) both vineyards (denoted as actions 0,0/10.01 or 11, respectively). Each matrix elements contains two comma-separated values corresponding to the payoffs obtained by winemaker 1 and 2 respectively, g stands for the cost of disease control in one vineyard, R1 and R2 stands for the reward for implementing disease control in each winemaker’s own vineyard. x represents the reward for implementing disease control in the vineyard that is not one’s own. The possible actions of agent 1 correspond to rows, while actions of agent 2 correspond to columns. Nash equilibria corresponding to cases 1.2.3 in the list shown in Section 5 of the paper are indicated in the table as corresponding superscripts. It is assumed that 0 ≤ x < g. For the case of R1 > g and R2 the action pair (0,0) corresponds to a Nash equilibrium where no agent can unilaterally improve their utility. For the case of R1 < g and R2 > g, the only Nash equilibrium corresponds to payments of winemaker 1 toward vineyard 1 and winemaker 2 toward vineyard 2 (payoff matrix element 10.01). The case of R1 > g and R2 < g corresponds to a Nash equilibrium of (10,00): winemaker 1 paying toward vineyard 1 and winemaker 2 choosing not to pay. As described in this paper, the payoffs R1 and R2 may contain non-financial and subjective components, for example corresponding to a high environmental motivation.

<table>
<thead>
<tr>
<th>Payments</th>
<th>00</th>
<th>01</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
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<td>x, R2 − g</td>
<td>R1, x − g</td>
<td>R1, x − g</td>
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<td>01</td>
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<td>x − g, R2</td>
<td>R1, x − g</td>
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<td>10</td>
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<td>R1, x − g</td>
<td>R1, x − g</td>
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<tr>
<td>11</td>
<td>R1, x − g</td>
<td>R1, x − g</td>
<td>R1, x − g</td>
<td>R1, x − g</td>
</tr>
</tbody>
</table>
Because this is an extreme even individual purpose game we can apply the importance of being different theorem: The only Nash equilibrium is that each winemaker is paying to control the disease in their respective vineyards and not contributing to keeping the vineyard of the competitor disease free. Alternatively, the existence of this unique Nash equilibrium can be seen in Table 4. Note that it is for this case not necessary to explicitly list all possible payoffs in order to identify Nash equilibria because one can instead apply the importance of being different theorem.

- Case 2: \( R_1 < g \) and \( R_2 < g \): The only Nash equilibrium is that no winemaker is paying for disease control. We can see that by listing the payoffs corresponding to all \( 4 \times 4 \times 4 \) possible action combinations of winemakers 1 and 2 as shown in Table 4.
- Case 3: \( R_1 > g \) and \( R_2 < g \): The only Nash equilibrium is that winemaker 1 is paying for disease control in his/her vineyard and winemaker 2 chooses to not pay. This can again be seen by consulting the payoff matrix shown in Table 4.

Although the results obtained here (namely Cases 1, 2, and 3) are consistent with the noncooperative game results as in Atallah et al. (2017), they are not driven by a difference in the economic value of the resource managed by each winemaker. Rather, they are driven by their motivation asymmetry, as defined in the importance of being different theorem, holding the value of the natural resource (wine quality) constant. Note that asymmetry for the example of winemakers can be more pronounced. For example, a producer of high-quality wines has a higher incentive to keep his vineyard disease free, for his loss of reputation would be more pronounced compared to a producer of low quality wine (corresponding to \( r_1 > r_2 \)). This goes beyond the scope of this paper; for more details, see reference Atallah et al. (2017).

6. Discussion

The results demonstrate that the ranking of a group with respect to goal achievement depends on the utilized scoring method as well as on the influence of additional challenges (an additional-defection challenge and a variable-load challenge were examined). The results suggest that (partial) polarization of a group can have advantages for robust, compartmentalized problem-solving of a group. This polarization is particularly interesting in connection with the achievement of goals, for which only a few (minority) of the group members are motivated to pay toward its solution. In such cases, previously reported approaches for reaching cooperation (such as coercion, kin-selection, costly punishment, reputation, reciprocity, etc.) do not readily apply (Fehr, 1999; Fehr and Gächter, 2000; Dohmen et al., 2006; Panchanathan and Boyd, 2004; Fowler, 2005). Indeed, such mechanisms that favor compliance with the expectations of the majority may work against someone who is attempting to exert individual efforts toward achieving a non-profit goal that is considered important only by a minority.

These results were obtained under the simplifying assumption that someone who is highly biased toward one goal does not attempt to hinder others in achieving a different goal. Also, one should be guarded against extrapolating the obtained results for groups with up to five members to groups of dramatically larger sizes. Furthermore, we know through reported experimental results that groups that do not possess means of communication between its members do not necessarily choose the “best” equilibrium point, nor do they choose equilibrium points with equal probability (Van Huyck and Battalio, 1990). The authors of the study suggest that this is so because participants may conclude that other players cannot be relied upon to choose the action corresponding to the highest-payoff equilibrium. This then leads some participants to conclude that choosing the action corresponding to the highest-payoff equilibrium is “too risky”.

From the results, it is tempting to speculate that the ubiquitous phenomenon of a left-wing/right-wing political spectrum might be understood as a special case in which the two commons to be maintained by a society represent a social network (helping others) and a social shield (protecting from others). This situation is called here the Great Tale of Two Commons (GTTC). The results shown in Tables 1–3 corresponding to groups of size 2, 3 and 4, respectively, indicate that polarized groups outperform groups consisting only of participants who value all goals equally. This suggests that groups whose political spectra are narrow (only moderates) are less engaged in voluntary efforts compared to semi-polarized groups that partially consist of participants who are biased toward one or the other goal.

In 2007, Dixit and Weibull published a model of voters that are given the choice of 5 different policies in order to better understand political processes and voter polarization (Dixit and Weibull, 2007). In their model, voters are subject to a learning process were failed policies are interpreted by non-centrist voters that their favored approach was not sufficiently adhered to, thus promoting more extreme political positions among them and leading to polarization. Conversely, successful policies lead in their model to a convergence to a state where all voters agree on a policy that is perceived to be optimal. The GTTC paradigm differs from that of Dixit and Weibull. Their model suggests that political polarization follows as a result of (i) a learning process and (ii) a consequence of failed policies (Dixit and Weibull, 2007). The GTTC and the Dixit and Weibull model may be falsifiable: in the GTTC model, years of successful policies involving relatively static demands on a society would lead to maintained polarization, whereas in the Dixit and Weibull model the same constraints would lead to convergence.

This strategy of compartmentalizing group-goals can be extended by adding further goals and respective member-specific motivations; thus leading to higher-dimensional achievement games, including the extreme case of an achievement game in which the number of goals is equal to the number of group members. The one and only equilibrium corresponds to the case in which each group member is uniquely motivated to reach one particular goal represents a fascinating case of goal achievement. Because the chosen actions of the group members differ, it serves an example of anti-coordination. As can be seen by the results for groups of three members faced with the challenge of achieving two goals (Table 2), the three strategies of coordination (exemplified by group “OOO”), anti-coordination (exemplified by group “AOB”) and prioritization (i.e. the reliable achievement of one goal combined with the ignorance of the other goals that are perceived to be less important, exemplified by groups “AAA” and “BBB”) all perform quite well. Top-performing larger groups can bring about combinations of these strategies, as exemplified by the “AABB” group of four members who succeed in achieving two goals by two members focusing on the first goal and the two other members focusing on the second goal (see Table 3). This scenario is similar to congestion or crowding games in which agents avoid making similar choices that could lead to the over-use of any one strategy (Rosenthal, 1973). In other words, one strategy to cope with the difficulty of coordination games may be to “change the game” such that the need for coordination is minimized.

Another example where a related strategy of problem-solving via high-dimensional anti-coordination appears to be used is parenting. Here, parents commit all necessary resources toward the upbringing of their own progeny while contributing comparatively little to the upbringing of those who are not part of their family.

Applying this approach to the environmental situation, one needs to remind oneself of survey results that consistently show
that environmental goals are viewed by the majority as less important than, for example, economic considerations (Upham, 2009). If goal 1 stands for a goal of “economic growth” and goal 2 stands for a goal of “environmental protection”, the represent-ative 4-member groups from Table 3 would thus be called “AAAA”, “AAAD” or “AABD”, in other words groups consisting mostly of members who would give economic growth priority over environmental protection. Such groups perform relatively well by succeeding reliably in achieving the prioritized goal while ignoring the goal viewed as less important (the ALL score of Table 3 shows that the fraction of equilibrium points in which both goals are achieved is in all three cases zero). The opportunity suggested by the research presented in this paper is now to use a less ambitious second goal that can be achieved by the minority (“B-agents”), for which the second goal has high priority. An example of a “less ambitious second goal” could be the protection of biodiversity hotspots in contrast to approaches that require the majority of a population to value environmental goals higher than economic goals. Just like voluntary firefighters solve a minority problem (firefighting) by taking on their own shoulders the burden of solving the problem they feel strongly about even though they did not cause it. This mode of environmental protection would suggest a new type of environmental protectionists, who solve environmental problems by personally taking on the burden of solving them. Such an approach would be fundamentally different and yet complementary to current “mainstream” environmental approaches that attempt to convince all participants to give economic and environmental considerations similar priority (which applied to Table 3 would be, if successful, leading to “O000” groups).

One may view businesses that voluntarily subject themselves to the 1% for The Planet initiative as a real-life example of such a “heroic effort” approach to protect nature (http://onepercentfortheplanet.org). In this initiative, participating businesses agree to donate at least 1% of their annual sales toward environmental causes. This amount is substantially higher compared to charitable contributions of average firms; also there are no explicit demands that all firms should donate this much. Such voluntary efforts alone may not be sufficient to protect the atmosphere or oceans, but they may help to solve difficult issues like the protection of land-based biodiversity hotspots.

Such novel approaches could be combined with other recent innovations, such as the augmentation to the “reduce, reuse, recycle” paradigm in order to reduce or even eliminate rebound effects (Bindewald, 2013). Rebound effects describe economic feed-back mechanisms that render reduction in resource usage less pronounced. For example, increased efficiency (say in transportation or in lighting technology) may lead to financial savings; if those financial savings due to efficiency gains are spent on average economic activities, the environmental benefits due to the superior technology may not be as pronounced as initially assumed. While the precise magnitude of rebound effects are subject of ongoing research, it has been thus far mostly overlooked that rebound effects are straight-forward to eliminate entirely. As shown in reference Bindewald (2013), if only 1% of financial savings obtained through efficiency gains are donated toward applicable environmental causes, a high estimate of rebound effects is more than eliminated. This approach of reinvesting financial savings toward nature protection has been termed Restore and can be linked to each of the sustainability efforts of reducing, reusing and recycling. If this approach is based on voluntary efforts, it will require motivated participants. The work in this paper explores theoretically how a motivated minority can accomplish goals that do not enjoy the support of the majority of a group. That is the rationale why it may be worth applying the theory developed in this paper toward the Restore approach.

The mechanism proposed here corresponds to an incentive for altruistic behavior in terms of member’s subjective utility function, not in terms of their objective burden. The results suggest that one of the strategies for achieving goals in a population may be to have a “noisiness” of motivations of its members toward multiple objectives, such that for each challenge a population faces, a subset of highly motivated members voluntarily emerges that personally takes on the burden of solving the challenge at hand. The results suggest, that differences in motivations and priorities with respect to life’s various challenges we might have with someone, are indeed an opportunity and the result of Nature’s approximate solution to escape mutual defection equilibria.

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